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Addendum to Classification of irreducible holonomies of torsion-free affine connections

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The real form $\mathrm{Spin}(6,\mathbb{H}) \subset \mathrm{End}(\mathbb{R}^{32})$ of $\mathrm{Spin}(12,\mathbb{C}) \subset \mathrm{End}(\mathbb{C}^{32})$ is absolutely irreducible and thus satisfies the algebraic identities (40) and (41). Therefore, it also occurs as an exotic holonomy and the associated supermanifold $\mathcal{M}_{\mathfrak{g}}$ admits a SUSY-invariant polynomial. This real form has been erraneously omitted in our paper.

Also, the two real four-dimensional exotic holonomies, whose occurrences were unknown at the time of writing, have been shown to exist very recently by R. Bryant [B].

With these corrections, Table 3 and the table in Theorem C should read as follows.

*The original article appeared in **150** (1999), 77–149.

Table 3: List of exotic holonomies

group G	representation V	restrictions/remarks
$\mathrm{T}_{\mathbb{R}} \cdot \mathrm{Spin}(5,5)$	$\mathbb{R}^{16}$	
$\mathrm{T}_{\mathbb{R}} \cdot \mathrm{Spin}(1,9)$	$\mathbb{R}^{16}$	
$\mathrm{T}_{\mathbb{C}} \cdot \mathrm{Spin}(10,\mathbb{C})$	$\mathbb{C}^{16} \simeq \mathbb{R}^{32}$	
$\mathrm{T}_{\mathbb{R}} \cdot \mathrm{E}_6^1$	$\mathbb{R}^{27}$	
$\mathrm{T}_{\mathbb{R}} \cdot \mathrm{E}_6^4$	$\mathbb{R}^{27}$	
$\mathrm{T}_{\mathbb{C}} \cdot \mathrm{E}_6^{\mathbb{C}}$	$\mathbb{C}^{27} \simeq \mathbb{R}^{54}$	
$\mathrm{T}_{\mathbb{R}} \cdot \mathrm{SL}(2,\mathbb{R})$	$\odot^3\mathbb{R}^2 \simeq \mathbb{R}^4$	
$\mathrm{SL}(2,\mathbb{C})$	$\odot^3\mathbb{C}^2 \simeq \mathbb{R}^8$	
$\mathbb{C}^* \cdot \mathrm{SL}(2,\mathbb{C})$	$\odot^3\mathbb{C}^2 \simeq \mathbb{R}^8$	
$\mathbb{R}^* \cdot \mathrm{Sp}(2,\mathbb{R})$	$\mathbb{R}^4$	
$\mathbb{C}^* \cdot \mathrm{Sp}(2,\mathbb{C})$	$\mathbb{C}^4 \simeq \mathbb{R}^8$	
$\mathbb{R}^* \cdot \mathrm{SO}(2) \cdot \mathrm{SL}(2,\mathbb{R})$	$\mathbb{R}^2 \otimes \mathbb{R}^2 \simeq \mathbb{R}^4$	
$\mathbb{C}^* \cdot \mathrm{SU}(2)$	$\mathbb{C}^2 \simeq \mathbb{R}^4$	
$H_\lambda \cdot \mathrm{SU}(2)$	$\mathbb{C}^2 \simeq \mathbb{R}^4$	
$H_\lambda \cdot \mathrm{SU}(1,1)$	$\mathbb{C}^2 \simeq \mathbb{R}^4$	
$\mathrm{SL}(2,\mathbb{R}) \cdot \mathrm{SO}(p,q)$	$\mathbb{R}^2 \otimes \mathbb{R}^{p+q} \simeq \mathbb{R}^{2(p+q)}$	$p+q \geqslant 3$
$\mathrm{Sp}(1) \cdot \mathrm{SO}(n,\mathbb{H})$	$\mathbb{H}^n \simeq \mathbb{R}^{4n}$	$n \geqslant 2$
$\mathrm{SL}(2,\mathbb{C}) \cdot \mathrm{SO}(n,\mathbb{C})$	$\mathbb{C}^2 \otimes \mathbb{C}^n \simeq \mathbb{R}^{4n}$	$n \geqslant 3$
E_7^5	$\mathbb{R}^{56}$	
E_7^7	$\mathbb{R}^{56}$	
$\mathrm{E}_7^{\mathbb{C}}$	$\mathbb{R}^{112} \simeq \mathbb{C}^{56}$	
$\mathrm{Sp}(3,\mathbb{R})$	$\mathbb{R}^{14} \subset \Lambda^3\mathbb{R}^6$	
$\mathrm{Sp}(3,\mathbb{C})$	$\mathbb{R}^{28} \simeq \mathbb{C}^{14} \subset \Lambda^3\mathbb{C}^6$	
$\mathrm{SL}(6,\mathbb{R})$	$\mathbb{R}^{20} \simeq \Lambda^3\mathbb{R}^6$	
$\mathrm{SU}(1,5)$	$\mathbb{R}^{20}$	
$\mathrm{SU}(3,3)$	$\mathbb{R}^{20}$	
$\mathrm{SL}(6,\mathbb{C})$	$\mathbb{R}^{40} \simeq \Lambda^3\mathbb{C}^6$	
$\mathrm{Spin}(2,10)$	$\mathbb{R}^{32}$	
$\mathrm{Spin}(6,6)$	$\mathbb{R}^{32}$	
$\mathrm{Spin}(6,\mathbb{H})$	$\mathbb{R}^{32}$	
$\mathrm{Spin}(12,\mathbb{C})$	$\mathbb{C}^{32} \simeq \mathbb{R}^{64}$	

Notation: $\mathrm{T}_{\mathbb{F}}$ denotes any connected Lie subgroup of $\mathbb{F}^*$,

$$H_\lambda = \left\{ e^{(2\pi i+\lambda)t} \mid t \in \mathbb{R} \right\} \subseteq \mathbb{C}^*, \quad \lambda > 0.$$

Table from Theorem C

Group G	Representation space	Group G	Representation space
$\mathrm{Sp}(n,\mathbb{R})$	$\mathbb{R}^{2n}$	E_7^5	$\mathbb{R}^{56}$
$\mathrm{Sp}(n,\mathbb{C})$	$\mathbb{C}^{2n}$	E_7^7	$\mathbb{R}^{56}$
$\mathrm{SL}(2,\mathbb{R})$	$\mathbb{R}^4 \simeq \odot^3\mathbb{R}^2$	$\mathrm{E}_7^{\mathbb{C}}$	$\mathbb{C}^{56}$
$\mathrm{SL}(2,\mathbb{C})$	$\mathbb{C}^4 \simeq \odot^3\mathbb{C}^2$	$\mathrm{Spin}(2,10)$	$\mathbb{R}^{32}$
$\mathrm{SL}(2,\mathbb{R})\cdot\mathrm{SO}(p,q)$	$\mathbb{R}^{2(p+q)}$, $p+q \geqslant 3$	$\mathrm{Spin}(6,6)$	$\mathbb{R}^{32}$
$\mathrm{SL}(2,\mathbb{C})\cdot\mathrm{SO}(n,\mathbb{C})$	$\mathbb{C}^{2n}$, $n \geqslant 3$	$\mathrm{Spin}(6,\mathbb{H})$	$\mathbb{R}^{32}$
$\mathrm{Sp}(1)\mathrm{SO}(n,\mathbb{H})$	$\mathbb{H}^n \simeq \mathbb{R}^{4n}$, $n \geqslant 2$	$\mathrm{Spin}(12,\mathbb{C})$	$\mathbb{C}^{32}$
$\mathrm{SL}(6,\mathbb{R})$	$\mathbb{R}^{20} \simeq \Lambda^3\mathbb{R}^6$	$\mathrm{Sp}(3,\mathbb{R})$	$\mathbb{R}^{14} \subset \Lambda^3\mathbb{R}^6$
$\mathrm{SU}(1,5)$	$\mathbb{R}^{20}$	$\mathrm{Sp}(3,\mathbb{C})$	$\mathbb{C}^{14} \subset \Lambda^3\mathbb{C}^6$
$\mathrm{SU}(3,3)$	$\mathbb{R}^{20}$		
$\mathrm{SL}(6,\mathbb{C})$	$\mathbb{C}^{20} \simeq \Lambda^3\mathbb{C}^6$		

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